Calc BC 6.1b Notes and Examples Name: Block: Seat:

Intro to Euler's Method (6.1b)

1. (2013 BC 5) Consider the differential equation $\dfrac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$

   (a) Find $\displaystyle\lim_{x\to 0} \frac{f(x)+1}{\sin x}$

   $\frac{0}{0}$ so by L'H $= \displaystyle\lim_{x\to 0} \frac{f'(x)}{\cos x}$

   $\frac{dy}{dx} = y^2(2x+2)$

   (0, −1)

   $f(0) = -1$

   $f'(0,-1) = (-1)^2(2(0)+2)$

   $= 2$

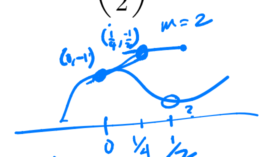
   tangent line $\quad y = 2(x-0)-1$

   So $f\left(\frac{1}{2}\right) \approx -1 + \frac{1}{2}(2) = 0$

   $f\left(\frac{1}{4}\right) = -1 + \frac{1}{4}(2) = -\frac{1}{2}$

   (b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

   Step Size ($\Delta x = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$ $\qquad \Delta x = \frac{1}{4}$

   $f(0) = -1$

   $f(1/4) \approx f(0) + \Delta y = (-1) + \left(\frac{1}{4}\right)(2) = -\frac{1}{2}$

   $f(1/2) \approx f(1/4) + \Delta y\Big|_{\left(\frac{1}{4}, -\frac{1}{2}\right)} = -\frac{1}{2} + (dx)\left(\left(-\frac{1}{2}\right)^2\left(\frac{5}{2}\right)\right)$ $\quad dx = \frac{1}{4}$

   $=$

   $\left(\frac{dy}{dx} = y^2(2x+2)\right)$

   $$\boxed{\frac{dy}{\cancel{dx}} = dx \cdot f'(x)}$$

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = 1$.

(on the exam $f(0) = -1$)

$$\frac{dy}{dx} = y^2(2x+2)$$

$$\int y^{-2}\,\frac{dy}{dx} = \int 2x+2 \quad dx$$

$$\frac{y^{-1}}{-1} = x^2 + 2x + C$$

$$\frac{-1}{y} = x^2 + 2x + C$$

$$-\frac{1}{1} = 0^2 + 2(0) + C \Rightarrow (C = -1)$$

$$\frac{1}{y} = -x^2 - 2x + 1$$

$$y = \frac{-1}{x^2 + 2x - 1}$$

2. Let $y = f(x)$ be the solutions to the differential equation $\frac{dy}{dx} = 2y - x$ with the initial condition $f(1) = 2$. What is the approximation for $f(0)$ obtained using Euler's method with two steps of equal length starting at $x = 1$?

   (a) $-\frac{5}{4}$

   (b) -1

   (c) $\frac{1}{4}$

   (d) $\frac{1}{2}$

   (e) $\frac{27}{4}$

$\Delta x = \frac{0-1}{2} = -\frac{1}{2}$

$f(1) = 2$

$f(\frac{1}{2}) \approx 2 + [\frac{-1}{2}](2(2) - (1)) = \frac{1}{2}$

$f(0) \approx \frac{1}{2} + [\frac{-1}{2}](2(\frac{1}{2}) - (\frac{1}{2})) = \frac{1}{4}$

3. Let $y = f(x)$ be the solutions to the differential equation $\frac{dy}{dx} = x - y - 1$ with the initial condition $f(1) = -2$. What is the approximation for $f(1.4)$ if Euler's method is used, starting at $x = 1$ with two steps of equal size?

   (a) -2

   (b) -1.24

   (c) -1.2

   (d) -0.64

   (e) 0.2

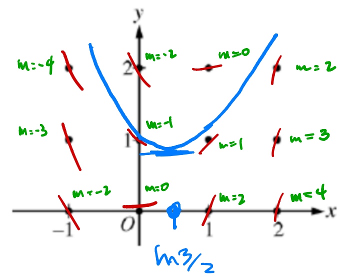
$\Delta x = \frac{1.4-1}{2} = \frac{.4}{2} = .2 = \frac{1}{5}$

$f(1) = -2$

$f(1.2) = -2 + \frac{1}{5}((1) - (-2) - 1) = -1.6$ (or $\frac{8}{5}$)

$f(1.4) = -1.6 + \frac{1}{5}((1.2) - (-1.6) - 1) = -1.24$

4. (2005 BC 4) Consider the differential equation $\frac{dy}{dx} = 2x - y$.



(a) On the axes provided, sketch a slope field for the given differential at the twelve points indicated, and sketch the solution curve that passes through the point (0,1).

(b) The solution curve that passes through the point (0,1) has a local minimum at $x = \ln(3/2)$. What is the y-value of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

(b) $\frac{dy}{dx} = 0 = 2x - y \Big|_{x = \ln 3/2}$

$0 = 2\ln 3/2 - y$

$y = 2 \ln 3/2$

$\left(\text{or } y = \ln(3/2)^2 = \ln 9/4 \right)$

(c) $f(0) = 1$ $\quad \Delta x = \frac{(-0.4 - 0)}{2} = -.2$

$f(-.2) = f(0) + (-.2)(2(0) - (1)) = 1 + .2 = 1.2$

$f(-.4) = f(-.2) + (-.2)(2(-.2) - (1.2)) = 1.2 + .32 = 1.52$

(d) $\frac{d}{dx}(2x - y)$

$2 - 1\frac{dy}{dx}$

$2 - (2x - y)$

$2 - 2x + y$